Common trends, Cointegration and Competitive Price Behaviour

John Hunter
Department of Economics and Finance, Brunel University, Uxbridge, Middlesex, UB8 3PH, England

Simon Burke
Economics Department
Reading University Business School

June 28, 2007

Abstract

The Article considers the impact of a single stochastic trend on the identification of competitive behaviour. The common trend measures the cumulated average response of the market to shocks in pricing behaviour. If the market definition is broad then all firms pricing ought to reflect this competitive urge. More specifically, individual prices will adjust in the long-run to the common trend to survive and this form of error correcting behaviour reveals a set of restrictions that econometrically identify the cointegrating vectors. Alternatively, non-competitive behaviour is observed either when there is more than one common trend or when a single price series drives the common trend. The latter case occurs when one of the price series is weakly exogenous for all the cointegrating vectors.

Key Words: Cointegration, Cointegrating Exogeneity, Common trend, Competition, Equilibrium Price Adjustment, Stochastic Trend, Weak Exogeneity.

JEL Classification: C32, D18, D40.
1 Introduction

In this article we define statistical criteria for determining competitive behaviour from the long-run decomposition of prices. Regulatory authorities and firms have exploited tests of stationarity and cointegration to influence regulatory procedure (Hunter and Ioannidis (2001)) and attempt to determine non-competitive behaviour ((Forni (2004), London Economics (2002)). Here, tests for stationary relative prices are seen as a special case of cointegration. The existing literature, addressing this issue through tests of cointegration has either considered small systems (La Cour and Møllgaard (2002), Hendry and Juselius (2001)) or sets of binary relations (De Vanya and Wals (1999)). When market prices are sufficiently inter-related in the long-run via cointegration, then the market is viewed as having a broad definition or being more competitive. It is our premise that for the case of multi-price comparisons, testing for stationarity amongst sets of price indices is best addressed by tests of cointegration, because multivariate cointegration tests are more appropriate when there are $n > 2$ prices and from a systems approach it is feasible to extend inference to distinguish between competitive and collusive behaviour.

In this article, we generalize the approach outlined by Hendry and Juselius (2001) to the case of multi-product price comparisons. For a competitive market with $n$ prices that are all integrated of order one (I(1)), then for a broad market definition $n - 1$ cointegrating relationships are required. The cointegrating relationships can be uniquely identified when all firms adjust to a single common trend. This may appear to be counter intuitive as price following is often seen as defining anti-competitive behaviour (Neumann (2001)). However, as is explained by Buccirossi (2006) the observation that prices are independently set is not necessarily consistent with behaviour that can be objectively described as being collusive. An important component of the econometric arguments that have been exploited to show that parallel pricing is linked to a broad market definition is dependent on the fact that price series are non-stationary and the price responses need to be proportional. Here, we demonstrate that competitive behaviour is only required to be proportional in the case where for all intents and purposes $n = 2$ as occurs for the conditional model in Hendry and Juselius (2001) and the sequence of pairwise comparisons associated with the model analyzed in De Vanya and Wals (1999)). While for $n > 2$ the observation of long-run parallel pricing could also be consistent with anti-competitive behaviour.\footnote{The approach discussed thus far relates only to the interaction of prices, though the models could be conditioned on other variables. Firstly, this is due to the fact that there is a significant literature that uses this methodology. Secondly, Forni (2004) explains that other methods, such as the residual demand approach and any analysis that draws on the calculation of elasticities may also be flawed as it does not account for both sides of the market.}

There is clearly some way to go before such an approach might define a test of competitiveness, but the analysis in Forni(2004) and London Economics (2002) study for the Dutch Telecommunications Regulator have already been used to associate a narrowly defined market with non-competitive behaviour. The current article is a reflection of the concerns voiced by Hunter (2003) as to the way the methodology has been applied.
with parallel pricing (Hendry and Juselius (2001)) and this proposition might be appropriately tested by determining whether the natural logarithm (log) of price proportions are stationary.

Here, \( n \) price responses are consistent with competitive behaviour when all prices have the same order of integration (I(1)), there are \( n - 1 \) cointegrating relationships or a single common trend and the common trend is driven by a combination of shocks to all \( n \) prices. The test of cointegration is a primary test of the proposition that all series are driven by a single common trend and thus a weighted average of the price shocks of all firms, but in the multiproduct case this does not of necessity imply parallel pricing. Parallel pricing only arises when \( n - 1 \) prices respond to a single price and this price is then weakly exogenous for the vector of cointegrating relationships (Johansen (1992)). In the latter case one firm price defines the stochastic trend and all firms respond to the prices set by that firm. The price that is weakly exogenous responds only to past values of that price and more generally to the shocks that apply to that firms price. In addition, the I(1) case considered here, can be extended to include variables that exhibit long-memory, as discussed by Robinson (2006)); the common stochastic trend is now replaced by a common strong attractor.

Firstly we consider the bivariate case and then the trivariate case. The trivariate case is an extension of the model analyzed by Hendry and Juselius (2001) and it is easy to show that the results considered here extend to the multi-variate case. The structural restrictions associated with a market where prices strongly interact uniquely identify the cointegrating vectors without further restrictions on the loadings and all prices are strongly inter-related in the long-run when each firms pricing behaviour is consistent with long-run correction to a single stochastic trend that excludes none of the other firms shocks. The former implies that the cointegrating rank is \( n - 1 \) and the latter that there are no weakly exogenous variables in the system.

2 The stochastic trend, Long-run Equilibrium Price Targeting (LEPT) and Cointegration.

Let us consider a market with \( n \) firms, all firms are viewed as being competitive firm when they all respond to a common stochastic trend and this common trend is driven by linear combinations of the shocks \( (\epsilon_i) \) that apply to all firms. This is an equilibrium price target (EPT) when each of the firms respond in the same way to this single common trend and the relationship between each firm price and this trend defines a specific set of restrictions on the \( n - 1 \) cointegrating relations \( (\beta) \) and such restrictions are sufficient to exactly identify \( \beta \).

\(^2\)Here we reflect on the information that derives from the long-run inter-action of prices, because we believe that in the long-run demand responses are more elastic and that arbitrage is more likely to force firms to respond to the forces of competition. However, there are other empirical methods directed at determining whether markets are competitive. Also see the discussion of variance in Abrantes-Metz, Freeb, Geweke and Taylor (2005). Hosken, O’Brien, Scheffman and Vita (2002) consider studies based on systems of demand equations to analyze
Competitive firms are viewed as correcting their price behaviour in response to some equilibrium price target. The underlying target to which the competitive firm responds is an average of the vector of all firms prices \( x_t = [p_{1t} \ldots p_{nt}] \) and for series that are all \( I(1) \) is defined by the non-stationary component of a single common trend.

**Definition 1** Let \( \exists p_t^* \) where:

\[
p_t^* = a'x_t = a'Cx_0 + a'C(\sum_{i=1}^{t} \epsilon_i + \mu)
\]

\[
+ a'\alpha(\beta'\alpha)^{-1} \sum_{i=0}^{\infty} (I + \beta'\alpha)^i \beta'(\epsilon_i + \mu)
\]

is a common trend or more generally a common trend plus stationary components that are non-unique, \( x_0 \) are initial values and \( \mu \) is the drift (see Johansen (1995) or Burke and Hunter (2005) for this definition). Then Long-run Equilibrium Price Targeting implies for \( p_{it} \sim I(d) \), \( \forall \ i = 1, \ldots n \), that:

\[
p_{it} - p_t^* = p_{it} - a'x_t \sim I(0).
\]

Where \( a' = [a_1 \ldots a_n] \). Therefore:

\[
p_{it} - p_t^* = p_{it} - a'x_t = (a'i_i - a')x_t = a'(i_i - i_n)x_t.
\]

If \( (i_i - i_n) = R_i \) then there are \( n \) cointegrating vectors of the form \( \beta_i = a'(i_i - i_n) \) that are dependent, when all prices have the same order of integration.

**Case 1**, bivariate models (Hendry and Juselius (2001), De Vanya and Walls (1999) and Forni (2004)), where \( a' = [a_1 \ a_2] \) and:

\[
\beta_n' = \begin{bmatrix}
\beta_{1,1} \\
\beta_{1,2}
\end{bmatrix} =
\begin{bmatrix}
a'R_1 \\
ad'R_2
\end{bmatrix} =
\begin{bmatrix}
a_2 & -a_2 \\
-a_1 & a_1
\end{bmatrix}
\]

where \( R_1 = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \), and \( R_2 = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \).

The appropriateness of Horizontal Mergers. And Froeb and Werden (1998) discuss the use of the difference in the Herfindahl Herschman index to assess welfare gains through mergers in homogenous product markets. While, Hunter, Ioannidis, Iossa and Skerratt (2001) have sought to provide measures appropriate to the analysis of consumer harm under conditions of imperfect information about price and quality.

Of course, in the case of merger, significant resources are often employed by anti-trust authorities to analyze a small number of cases. Which is the basis of this quest for more effective techniques as can seen by the significant interest in consumer harm amongst OECD countries and the European Union, Competition Commission. In this light, the degree of consumer detriment, as measured on the basis of survey data by the OFT (2000) and at the level of companies by Pierce-Brown and Graham (2001), would also appear to suggest that this is a significant issue.
After normalization
\[ \beta'_n = \begin{bmatrix} \frac{1}{a_1} & -\frac{a_2}{a_1} \end{bmatrix}. \]

Selecting \( n - 1 = 1 \) cointegrating vectors from the \( \beta'_n \):
\[ \beta' = \begin{bmatrix} 1 & -\frac{a_2}{a_2} \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}. \]

The cointegrating vectors take the form \( \rho_{1t} - \rho_{2t} \sim I(0) \) either when the price proportion is tested for stationarity as is suggested by Forni (2004), or from the cointegrating rank of the bivariate system as suggested by De Vanya and Walls (1999) or conditional on the price of oil as is the case in Juselius and Hendry (2001).

However, such tests of bivariate relationships are not appropriate when there are more than two price combinations as is the case in Forni (2004), De Vanya and Walls (1999) and London Economics (2002). This point was addressed by Hunter (2003) in his response to London Economics (2002). This move us to cases where \( n \) exceeds 2.

Case 2, The trivariate system with \( a' = [a_1 \ a_2 \ a_3] \):
\[ \beta'_n = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} a'R_1 \\ a'R_2 \\ a'R_3 \end{bmatrix} = \begin{bmatrix} a_2 + a_3 & -a_2 & -a_3 \\ -a_2 & a_1 + a_3 & -a_3 \\ -a_2 & a_1 & a_1 + a_2 \end{bmatrix}, \]
where
\[ R_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{and} \quad R_3 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \]

That after normalization
\[ \beta''_n = \begin{bmatrix} \frac{1}{a_1 + a_2} & -\frac{a_2}{a_1 + a_2} & -\frac{a_3}{a_1 + a_2} \\ \frac{a_2}{a_2 + a_3} & \frac{1}{a_2 + a_3} & \frac{a_3}{a_2 + a_3} \end{bmatrix} 3 \]
\[ \text{And det} \begin{bmatrix} \frac{1}{a_1 + a_2 + a_3} & -\frac{a_2}{a_1 + a_2 + a_3} & -\frac{a_3}{a_1 + a_2 + a_3} \\ -\frac{a_1}{a_1 + a_2 + a_3} & \frac{1}{a_1 + a_2 + a_3} & -\frac{a_3}{a_1 + a_2 + a_3} \end{bmatrix} = \frac{a_2^2 + a_1 a_2 + a_1 a_3 + a_2 a_3}{a_1 + a_2 + a_3} + \frac{a_2 a_3}{a_1 + a_2 + a_3} \left( -\frac{1}{a_1 + a_2 + a_3} - \frac{a_3}{a_1 + a_2 + a_3} \right) = 0. \]

Therefore the \( n = 3 \) potential cointegrating vectors are by definition dependent and all the prices cointegrate with the common trend.
Now consider, \( n - 1 \) cointegrating vectors and for convenience we will select for the non-normalized case:

\[
\beta' = \begin{bmatrix}
\beta_{11} & \beta_{21} & \beta_{31} \\
\beta_{12} & \beta_{22} & \beta_{32} \\
\end{bmatrix} = \begin{bmatrix}
a_2 + a_3 & -a_2 & -a_3 \\
-a_1 & a_1 + a_3 & -a_3 \\
\end{bmatrix}.
\]

It follows from Bauwens and Hunter (2000) that a sufficient\(^4\) condition for the generic identification of a particular long-run structure from a set of cointegrating relations is the existence of an \( r \) dimensional matrix \( B \) that is invertible. There are \( \frac{r!}{2!(r-2)!} = 3 \) unique combinations that can be used to normalize the system and thus yield an identification. Consider, the orientation that operates on the first two columns of \( \beta' \):

\[
B_{1,2} = \begin{bmatrix}
a_2 + a_3 & -a_2 \\
-a_1 & a_1 + a_3 \\
\end{bmatrix}.
\]

We require for an appropriate orientation that:

\[
\det(B_{1,1}) = \det \left( \begin{bmatrix}
a_2 + a_3 & -a_2 \\
-a_1 & a_1 + a_3 \\
\end{bmatrix} \right) = (a_1 + a_2 + a_3)a_3 \neq 0.
\]

It follows that under price homogeneity or the condition \( a'\iota = 1 \), identification follows from

\[
(a_1 + a_2 + a_3)a_3 = (a'\iota)a_3 = a_3.
\]

Hence, under LEPT, given price homogeneity \( (a'\iota = 1) \) the system is identified when \( a_3 \neq 0 \).\(^5\)

**Remark 2**

\[
\beta' = \begin{bmatrix}
a_2 + a_3 & -a_2 & -a_3 \\
-a_1 & a_1 + a_3 & -a_3 \\
\end{bmatrix}
\]

is identified when there exists \( B_{i,j} \) for \( i, j = 1, 2; 1, 3; 2, 3 \) such that \( \det(B_{i,j}) \neq 0 \). This result is consistent with LEPT when \( \det(B_{i,j}) = a_3 \).\(^6\)

A further necessary condition for empirical identification relates to Theorem 3 in Boswijk (1996) and as applied by him, confirms an appropriate normalization. Here, it implies that the orientation selected has a long-run reduced

\(^4\)It is sufficient rather than necessary, because there are a number of such orientations that may apply.

\(^5\)Notice that so far we are assuming that \( a \neq \alpha_+ \). Though the data may often be transformed such that \( a \approx \alpha_+ \).

\(^6\)Notice that according to Boswijk(1996) theorem 2 a necessary condition for the identification of

\[
\begin{bmatrix}
a_2 + a_3 \\
-a_1
\end{bmatrix}
\]

is \( \text{rank} \left( \begin{bmatrix}
-a_2 & -a_3 \\
a_1 + a_3 & -a_3
\end{bmatrix} \right) = 2. \)
form and it follows from the normalization rule that $\beta'$ can thus be empirically identified for the $i, j$ ordering when $\det(B_{i,j}) \neq 0$ and:

$$\beta'_{ij} = \begin{bmatrix} I_{n-1} & B_{i,j}^{-1}b_{\neq ij} \end{bmatrix}.$$ 

For the long-run reduced form to be non-trivial for the orientation $i, j$, $b_{\neq ij} \neq 0$. If $b_{\neq ij} = 0$, then one of the prices is long-run excluded and when $r = n - 1$, then this implies $\beta'_{ij} = \begin{bmatrix} I_{n-1} & 0 \end{bmatrix}$ and this for $r = n - 1$ contradicts the notion that all the series are $I(1)$.

Hence for generic and empirical identification of $\beta'$ we require one ordering such that:

$$\det(B_{i,j}) = \det\left(\begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix}\right) = \beta_{11}\beta_{22} - \beta_{12}\beta_{21} \neq 0$$

and

$$b_{\neq ij} = \begin{bmatrix} \beta_{31} \\ \beta_{32} \end{bmatrix} \neq 0.$$ 

The identification of a long-run system $(\beta)$ that arises from LEPT, implies that in addition to any normalization of an equation, the order condition necessary, but not sufficient for generic identification, $r^2 - r = 2$ (Burke and Hunter, 2005, Chapter 5) must hold. Hence, for some normalization:

$$\beta_{31} - \beta_{32} = 0 \text{ and } \beta_{11}\beta_{22} - \beta_{12}\beta_{21} + \beta_{32} = 0,$$

are necessary for generic identification. The sufficient condition for generic identification implies a rejection of non-identification that occurs when:

$$\beta_{32} \neq 0.$$ 

At this point we can complete the generic identification via the imposition of two normalizations or the application of two additional restrictions. Here generic identification is obtained from the imposition of the homogeneity restriction:

$$\beta_{11} + \beta_{21} + \beta_{31} = 0 \text{ and } \beta_{12} + \beta_{22} + \beta_{32} = 0.$$ 

For $n$ companies

$$\forall i = 1, 2, \ldots n, \sum_{j=1}^{n} \beta_{ji} = 0.$$ 

It easy to show that that these restrictions are consistent with $p_{it} - p_{t}^\ast \sim I(0)$ for $i = 1, 2, \ldots n$, and $a' = 1$.

The above result can be readily generalized to the case where there are more than three prices and from LEPT that enough additional restrictions apply to the cointegrating vectors.

If now we consider the special case where $a = \alpha_1$, $a_1' \alpha = 0$ and $\text{rank}(\alpha) = n - 1$, then rather than approximating the common trend, $p_{t}^\ast = \alpha_1' x_t$ when it
is possible to remove the initial condition by sequential demeaning each price series in turn (Taylor, 1999). If \( a' = \alpha'_C \), \( C = \beta_{1} (\alpha'_C \beta_{1})^{-1} \alpha'_C \), \( \beta'_{1} \beta = 0 \), then we can isolate the trend component by multiplying (1) by \( \alpha'_C \). Therefore:

\[
\alpha'_C x_t = \alpha'_C C x_0 + \alpha'_C C \left( \sum_{i=1}^{t} \epsilon_i + \mu \right) + \alpha'_C \alpha (\beta' \alpha)^{-1} \sum_{i=0}^{\infty} (I + \beta' \alpha)^i \beta' (\epsilon_i + \mu)
\]

\[
= \alpha'_C x_0 + \alpha'_C \left( \sum_{i=1}^{t} \epsilon_i + \mu \right).
\]

Setting the initial condition to zero it follows that the common trend is defined by:

\[
\alpha'_C x_t = \alpha'_C \left( \sum_{i=1}^{t} \epsilon_i + \mu \right).
\]

Now replacing \( a' \) by \( \alpha'_C \) in the trivariate system \( \alpha'_C = \begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \end{bmatrix} \) and after normalization there are \( n - 1 \) cointegrating vectors:

\[
\beta' = \begin{bmatrix}
1 & \beta_{21} & \beta_{31} \\
\beta_{12} & 1 & \beta_{32} \\
\frac{-\alpha_{11}}{\alpha_{11} + \alpha_{31}} & \frac{-\alpha_{21}}{\alpha_{21} + \alpha_{31}} & \frac{-\alpha_{31}}{\alpha_{11} + \alpha_{31}}
\end{bmatrix}
\]

The \( n - 1 \) restrictions implicit in \( \beta \) that exactly identify the cointegrating vectors are the same restrictions as occur when \( p_i^t \) is not exactly equivalent to \( \alpha'_C x_t \).

It follows that competitive behaviour is rejected when there are less than \( n - 1 \) cointegrating vectors as a group of firms may be following an alternative common trend and as a result they may not be responsive to the competitive price. The latter can occur when some of the prices are of another order of integration, but this would imply a different cointegrating rank. This leads to the following result:

**Corollary 3** Competitive behaviour requires a similar price response so it follows for LEPT we require \( p_i \sim I(d), \forall i = 1, \ldots, n \).

**Proof.** If there are \( j \) prices such that \( p_i \sim I(d_2), \forall i = 1, \ldots, j \) and \( d_2 > d \), then it follows that \( \text{rank}(\Pi) \leq n - 1 \). If \( d_2 = 2 \), then there are \( r_1 = j - 1 \) cointegrating vectors \( \beta_j \) implying \( \text{rank}(\Pi) \geq j - 1 \) when the \( j \) prices satisfy the restriction associated with LEPT. But when the other \( n - j \) prices also satisfy LEPT, then \( \text{rank}(\Pi) \geq n - j - 1 \). It follows by combining the two minimum rank conditions, that \( \text{rank}(\Pi) \leq n - j - 1 + j - 1 \). Hence, when prices are of different orders of integration they can best follow two independent random walks, meaning the market is not competitive.

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7Where from the definition of the Vector Error Correction Model (VECM), \( \Pi = \alpha \beta' \), \( \text{rank}(\Pi) = r \) and for cointegration in the \( I(1) \) context \( r < n \).

8If \( d_2 = 0 \), then there are \( j \) inter-related prices and the system is separated into two blocks.
It follows from Corollary 3 above that LEPT requires all series to be of the same order of integration otherwise different market segments may respond to different trends as \( \text{rank}(\Pi) \leq n - 2 \) and this violates Definition 1. It follows from Definition 1, \( \text{rank}(\Pi) \leq n - 1 \) and for LEPT that all prices cointegrate with the common trend. However, this type of relation is only consistent with competitive behaviour when the cointegrating relations depend on all prices or we preclude the case where, by any simple re-ordering, \( \alpha_{1}^{\top} = \begin{bmatrix} 0 & 0 & \alpha_{3,1} \end{bmatrix} \).

More specifically the identifying cointegrating combination negates the possibility that \( n - 1 \) prices depend exactly on a single price; this is the case where one of the prices is long-run weakly exogenous and all prices react to this price. More specifically, LEPT relates to a single price leader that does not respond to any other prices or in the simplest system this price follows a random walk and all prices respond to this form of price leadership. Hence, LEPT is necessary, but not sufficient for a market to be competitive as we also require none of the price variables to be weakly exogenous. If a price is weakly exogenous then the common trend is purely driven by the shocks to that firms price and do not respond to the market circumstances of the other firms.

We will show next that any violation of the condition \( \text{rank}(\Pi) = n - 1 \) will lead to prices following different trends, while weak exogeneity (WE), cointegrating exogeneity (CE), or sub-system long-run exogeneity (SSLE) all lead to a partial or complete failure of competitive pricing.

3 Exogeneity, Rank \( n - 2 \) and narrow market behaviour.

Firstly, we consider the case where there are \( n - 1 \) cointegrating vectors, but a single price is weakly exogenous for the cointegrating vectors (Johansen (1992)). Then we will consider further decompositions of the cointegrating vectors associated with CE (Hunter (1992)).

such that:

\[
\Pi = \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,1} & \Pi_{2,2} \end{bmatrix} = \begin{bmatrix} \alpha_{1} \\ \alpha_{2} \end{bmatrix} \begin{bmatrix} \beta_{1} \\ \beta_{2} \end{bmatrix}'
\]

\[
= \begin{bmatrix} \alpha_{1}\beta_{1}' & \alpha_{1}\beta_{2}' \\ \alpha_{2}\beta_{1}' & \alpha_{2}\beta_{2}' \end{bmatrix}
\]

It follows from the differential in the order of integration that there are \( n - j - 1 \) cointegrating relations amongst the higher order variables that are \( I(0) \). Therefore, without loss of generality \( \Pi_{2,1} = 0, \text{rank}(\Pi_{2,2}) \leq n - j - 1 \) and for ease of exposition, we set \( \Pi_{1,2} = 0 \):

\[
\Pi = \begin{bmatrix} \Pi_{1,1} & 0 \\ 0 & \Pi_{2,2} \end{bmatrix} = \begin{bmatrix} \alpha_{1}\beta_{1}' & 0 \\ 0 & \alpha_{2}\beta_{2}' \end{bmatrix}
\]

If we now consider the cointegrating relations associated with \( \beta_{1}' \) there are \( j \) homogeneity restrictions that apply to these price series and For ease of exposition, we set \( \Pi_{1,2} = 0 \) and

\[
\sum_{k \neq i} \beta_{ki} = -1 \quad \text{and as a result} \quad \text{rank}(\Pi_{1,1}) = j - 1.
\]

Therefore, LEPT implies that \( \text{rank}(\Pi) \leq n - 2 \).
If there are \( n-1 \) cointegrating vectors then \( \alpha \) is an \( n \times r \) matrix of loadings. It follows from Johansen (1992) that for WE of a variable for the parameters of interest \( (\beta) \), then a row of \( \alpha \) is set to zero. With \( \text{rank}(\alpha) = n - 1 \), then only one price can be weakly exogenous, otherwise \( \text{rank}(\alpha) < n - 1 \) and there are fewer than \( n - 1 \) cointegrating relations and the condition that there is a single common trend is violated.

If there is a single common trend, a single weakly exogenous variable and \( a' = a'_{\perp} \), then from the condition \( aa'_{\perp} = 0 \), it follows without loss of generality that \( a'_{\perp} = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \) and this implies that the \( n^{\text{th}} \) price defines the common trend. More generally, when \( a_i = 0 \) then the equation driving variable \( i \) contains no long-run components and this means that it follows a generalized random walk or this series defines the stochastic trend solely driven by the residual miss-pricing associated with that market segment alone.

**Corollary 4** If \( \text{rank}(\beta) = \text{rank}(\Pi) = n - 1 \) and \( \alpha' = \begin{bmatrix} \alpha'_{n-1} & 0 \end{bmatrix} \) for some ordering of the \( p_i \) \( i = 1, \ldots, n \), then this implies a broad market as all prices interact, but non-competitive behaviour as all prices follow \( p_n \).

**Proof.** In general, \( a'_{\perp} = \begin{bmatrix} \alpha_{1\perp} & \alpha_{2\perp} & \cdots & \alpha_{n\perp} \end{bmatrix} \) and the common trend drives all prices.

\[
p^*_t = a'x_t = a'_{\perp}x_t = \alpha'_{\perp}x_0 + \alpha'_{\perp}(\sum_{i=1}^{t} \epsilon_i + \mu).
\]

While in the WE case:

\[
\alpha' = \begin{bmatrix} \alpha'_{n-1} & 0 \end{bmatrix}
\]

implies that the normalization is unique as it is the only normalization by which \( p_i \) for \( i = 1, \ldots, n \) is conditioned on an exogenous variable \( p_n \). It follows from price homogeneity that under WE, \( \alpha'_{\perp} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \) and from the definition of a common trend:

\[
\begin{bmatrix} 0 & \cdots & 1 \\ \vdots \\ p_{nt} \end{bmatrix} = p_{nt} = p_{n0} + \sum_{i=1}^{t} (\epsilon_{ni} + \mu_n).
\]

For simplicity we consider the trivariate case and under LEPT:

\[
\beta' = \begin{bmatrix} \alpha_{2\perp} + \alpha_{3\perp} & -\alpha_{2\perp} & -\alpha_{3\perp} \\ -\alpha_{1\perp} & \alpha_{1\perp} + \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix} = \begin{bmatrix} \alpha_{3\perp} & 0 & -\alpha_{3\perp} \\ 0 & \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix}.
\]

Therefore:

\[
\beta'x_t = \begin{bmatrix} \alpha_{3\perp} & 0 & -\alpha_{3\perp} \\ 0 & \alpha_{3\perp} & -\alpha_{3\perp} \end{bmatrix} \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} = \begin{bmatrix} \alpha_{3\perp}(p_{1t} - p_{3t}) \\ \alpha_{3\perp}(p_{2t} - p_{3t}) \end{bmatrix}.
\]
From price homogeneity, $\alpha_{n} = 1$. By definition $p_{t}^{n} = p_{nt}$ and all prices are driven by the stochastic behaviour that underlies $p_{nt}$. As $\alpha'_{1} = \begin{bmatrix} 0 & 0 & \ldots & 1 \end{bmatrix}$ this is without reference to the shocks that underlie the other firms prices.

If all firms prices are conditioned on $p_{nt}$, then firm $n$ is the long-run price leader and should the restriction in theorem 1 apply and $p_{t}^{n} = p_{nt}$; then equilibrium price targeting implies that:

$$\beta = \begin{bmatrix} I_{n-1} \ -\iota_{n-1} \end{bmatrix} \text{ and } \iota'_{n-1} = \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}.\quad(9)$$

Therefore, we have a broad market in the sense that firms follow the common trend, but when the common trend is driven by a single firm without reference to other firms or more pertinent without reference to the the direct shocks associated with miss-pricing by these other firms, then the firm must hold a dominant position in the market place or that firm must define the barometer to which all other firms respond. However, a barometer should not normally behave without reference to the other firms. It follows, with one price being weakly exogenous for the parameters of interest, that the $n^{th}$ firms price can be viewed as driving all the other firms prices. This, we would argue is a form of price leadership as the long-run is conditioned on the behaviour of the $n^{th}$ firm price. In this case, under the restrictions associated with LEPT all firms respond to those of the $n^{th}$ firm, but in the long-run the $n^{th}$ firm does not respond to any of the other firms prices. Hence, although there are $n-1$ long-run price relations and $\beta$ satisfies the right restrictions this is not a competitive case. Hence, for competitive behaviour, we have a further requirement that the common trend is not defined by a single firms price or that none of the prices are weakly exogenous for $\beta$.

Notice that LEPT also precludes the existence of a block triangular form to the long-run parameter matrix associated with CE (Hunter (1992)) as this implies a sub-block of equations that do not respond to the other firms prices. If $\Pi = \alpha \beta'$, then decomposing $\alpha$ and $\beta$, into $\alpha_{1} \text{ an } n_{1} \times r \text{ sub-matrix, } \alpha_{2} \text{ an } n_{2} \times r \text{ sub-matrix, } \beta_{1} \text{ an } n_{1} \times r \text{ sub-matrix and } \beta_{2} \text{ an } n_{2} \times r \text{ submatrix, then:}$

$$\Pi = \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,1} & \Pi_{2,2} \end{bmatrix} = \begin{bmatrix} \alpha_{1} \\ \alpha_{2} \end{bmatrix} \begin{bmatrix} \beta_{1} \\ \beta_{2} \end{bmatrix}' = \begin{bmatrix} \alpha_{1}\beta'_{1} & \alpha_{1}\beta'_{2} \\ \alpha_{2}\beta'_{1} & \alpha_{2}\beta'_{2} \end{bmatrix}.$$ 

The the primary condition for CE is $\Pi_{2,1} = 0$ and without loss of generality this implies $\alpha_{2} = \begin{bmatrix} 0 & \alpha_{11} \end{bmatrix}$ and $\beta = \begin{bmatrix} \beta_{1,1} & 0 \end{bmatrix}$. But this condition imposes restrictions on $\beta$ that are not consistent with LEPT as $\beta_{1,2}$ is normally not zero. As according to the CE case the second block of equations do not respond to behaviour in the first block of firms prices. What is occurring here, is that some firms operate without reference to other firms in the group, or they cointegrate and all the other firms long-run price responses do not respond to them. Hence one firms prices can be viewed as causing the other prices in the
system and the long-run forecasts of the first block of firms are conditioned on those of the second block. This is long-run non-causality and this behaviour, due to the block triangularity, is also reflected in the corrections that occur in the short-run. Hence, the short-run for the first \( n_1 \) firms responds to the long-run corrections of all firms, while the long-run of the the last \( n_2 \) firms only responds to the correction in those firms alone.

In the case of WE for \( \beta_1 \) (first block of beta) of \( x_2 \), then sub-system WE (SSWE) implies that a sub-block of firms may operate as a cartel, while the remaining firms operate as a following fringe. The sub-block cartel is non-competitive, while the other block are ‘innocent followers’. The condition for SSWE is:

\[
\alpha_{1,2} - \Sigma_{1,2}(\Sigma_{2,2})^{-1}\alpha_{2,2} = 0.
\]

This occurs when either \( \alpha_{1,2} = \Sigma_{1,2}(\Sigma_{2,2})^{-1}\alpha_{2,2} \) or \( \alpha_{1,2} = 0 \) and \( \Sigma_{1,2} = 0 \). This implies that the long-run associated with the first block of equations can be estimated without reference to the information content of the second block of equations.

For cases where \( \text{rank}(\beta) = n - 2 \), this may occur when there is a further rank deficiency. For example, where:

\[
\beta_{i,i} = \beta_{j,j}
\]

for some \( i \neq j \). This is matched price following (MPF), but since for LEPT \( \text{rank}(\Pi) = n - 1 \), then for this type of behaviour to exist \( \text{rank}(\Pi) < n - 1 \).

A number of side issues arise from there being at least two common trends. Firstly, individual prices may follow linear combinations of the common trends that happen to be different. In this case, one trend may eventually dominate. They follow the same linear combination (consistent with equilibrium) so there exists a unique \( p^* \), but \( \text{rank}(\Pi) < n - 1 \). They may also follow different linear combinations of the common trends that are not consistent with equilibrium as such divergence of prices implies death or dominance.

4 Conclusion

In this article we considered the conditions required for competitive behaviour. We find that pricing is consistent with competitive behaviour when: i) there are \( n - 1 \) cointegrating relationships, ii) the restrictions associated with LEPT are satisfied, iii) there does not exist an \( \alpha_i = 0 \) for all \( i = 1, ..., n \). Forni(2004) and Hendry and Juselius (2001) also require (i) and (iii), but beyond the bivariate case the restrictions associated with LEPT are not in general simple parity conditions. This has the further implication that tests of stationarity in an \( n \) firm system will not be appropriate.

Although we have concentrated on the case where there are stochastic trends, the same conditions apply when there is drift and more generally for higher order series when there is balanced cointegration. It is our conjecture that the
restrictions are still relevant for long memory processes with fractional cointegration. In particular, Robinson and Yajima (2002) consider the estimation of fractional cointegration amongst oil prices that turns out to be an n-1 case and Robinson (2006) presents a very general expression of cointegration for stationary long memory processes that cointegrate. Hence, the results presented here are not limited to cases where prices are I(1).

5 References


submitted to the Nederlandse Mededingingsautoriteit, August 2003


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